On the Approximate Solution of Complex Combat Games

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We view air-to-air combat as a two-target differential game between armed opponents. Although it is presently beyond the state of the art to solve for the optimal controls for both of the opponents, we can constrain the game to searching for each initial condition for the "best" guidance law for each opponent from the sets of those considered. This constraint changes evaluation of the game to that of a discrete game, which can be solved by enumeration. As an example, a one-on-one two-dimensional air-to-air combat game has been designed. The game includes both attack and evasion guidance laws for both aircraft. To decide which of the guidance laws to use, the quantized initial-condition space is flooded with four simulated fights for each initial condition, depicting the strategies for the two opponents: attack–attack, attack–evasion, evasion–attack, and evasion–evasion, and the outcomes are recorded. For each initial condition, the minimax procedure from discrete games selects the best choice from the available strategies for both players. The results are feedback and outcome charts that can be used to turn one of the automatic opponents into an intelligent opponent against a human pilot.

Nomenclature

ATA	= radar antenna train angle = angle from aircraft x-body axis to opponent
C_{d0}, k_d, C_{brake}	= drag coefficients functions of Mach number
D	= drag
g	= gravitational acceleration
k, K	= guidance law gains
n, n_c	= actual and commanded load factors, + right turn − left turn
r, b, m	= subscripts indicating red fighter, blue fighter, and missile
R	= range between fighters
S	= wing surface area
T	= thrust
T_r, T_b	= track angles, counterclockwise angles from relative position vector to aircraft velocity vector
v, v_c	= actual and commanded aircraft speed (no winds considered)
W	= aircraft weight
x, y, z	= Cartesian coordinates of aircraft position
ψ	= heading angle, 0 along x axis
ρ	= air density

Introduction

THEORETICAL air-combat studies using the differential-game approach deal with highly idealized environments, as well as other simplifications, such as role assignments (i.e., evade or attack).[1-5] A combat pilot cannot benefit directly from studying the results of such work. The singular-perturbation approach allows the use of more realistic aircraft models than had been used before.[6] However, these studies kept role assignments. Very few studies considered the problem of air combat between armed opponents.[7-9] Although providing new understanding, these theories are too complex for near-term deployment in air combat.

On the other hand, simulated air combat, manned or unmanned, is capable of any amount of complexity and realism. An example is the adaptive maneuvering logic program (AML),[10,11] which performs well against pilots as measured by comparison of the number of seconds the AML had angular advantage over the piloted aircraft and vice versa.[12] But determining if a change in tactics has made an improvement in exchange rate has been difficult.[13] The approach given here, which uses simulation as a primary means to gather data, can use the outputs from partial game theoretical studies and integrate them into a total air-combat evaluation.

To bridge the gap between theory and simulation, we combine the methods of simulation with a discrete differential game to study air combat. As a simple example, we are studying a two dimensional one-on-one air combat game, where both aircraft have radar information of each other. The primary purpose of this game is to understand the complex issues of air combat in a game that is realistic enough to include most of the features of real air combat without getting lost in the complexities of the third dimension. A secondary reason for choosing a two-dimensional game was that, in order to obtain pilot participation, the computations had to be performed in real time, and the display had to be adequate for the pilot to control the aircraft. This had to be accomplished within the limitations of a Sun 2 workstation.

Both aircraft used in this study are modeled after the F-15 fighter, using Mach- and altitude-dependent drag and lift tables and realistic g limits. To study combat between unequal aircraft, either aircraft may be changed from the norm by reducing maximum thrust, g limits, or via the weight or the weapon's envelope. One aircraft, red, is always flown automatically by a small set of guidance laws that are switched by certain conditions. The other aircraft, blue, may be controlled by a human pilot or be flown automatically. In the present paper, results are given for the automatic opponents only. The manual modes were used by the author and a test pilot to eliminate the more obvious flaws of the automatic system. The automatic guidance laws were inferred from rather vague descriptions given in pilot's tactics manuals and books.[14-16]

When both aircraft are in an automatic mode, we treat the outcomes of the game as those of a deterministic discrete game. By considering all combinations of attack and evasion modes for both aircraft for many initial conditions, we can map out regions of combat outcomes for the case where both automatic pilots use the most favorable tactics implemented in a guidance law, either attack or evasion, for the given initial

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condition. This requires at least four experiments for each initial condition and application of the min-max procedure of discrete games to find the appropriate guidance law for both opponents. A combined map of all outcomes gives insight into which regions of the initial-condition space a fighter should attempt to be before being detected by the opponent. Stored choices of guidance laws over a large set of initial conditions can provide feedback charts that may be used to simulate a knowledgeable opponent in the sense that he/she knows when and how to attack or evade for any given situation. The game ends when one missile hits and no other missile has been launched, when both missiles hit for mutual destruction, or, when a specified time has been exceeded.

To understand the limits of this study, let us review the five phases of air combat[16]:

1) *Detection*. "He who sees, wins." It is important to see the enemy first, whether visually or electronically.

2) *Closing*. Four out of every five victims are taken by surprise. It is important to achieve an attacking position undetected, but also be on the lookout for other opponents who may surprise you.

3) *Attack*. If surprise is maintained, the attack is successful, provided the weapons work correctly. If the attack fails, the next phase is maneuver.

4) *Maneuver*. This is the phase in which the opponents are aware of each other, and in which the remaining fifth of kills are scored.

5) *Disengagement*. Basically, the work reported on here covers only phases 4 and 5 of air combat. However, it also provides a method to determine regions that should be achieved undetected in phases 2 and 3 for an easy win. This is accomplished by storing outcome charts along with the feedback charts.

Air Combat as a Differential Game

We can look at air combat as a two-target differential game.[4] Here each player has a target set and attempts to drive the state into his/her own target set without first being driven into the target set of the opponent. Thus, combat between players ("red" and "blue") can terminate in four ways with the performance ranking given in parentheses: blue wins (3), draw (2), joint kill (1), and red wins (0). The set of numerical ranking may be called blue's payoff values, which blue tries to maximize. If red has many more aircraft and pilots than blue, he/she may prefer a joint kill to a draw. In this case, red's payoff values would be identical to blue's, which red would try to minimize. This, then, would be a zero-sum differential game. If red, however, also prefers a draw to a joint kill, in red's view the payoff values for draw and joint kill would be reversed, and a nonzero-sum game results. These ideas are summarized in Table 1. The outcome numbers in Table 1, which agree with blue's payoff values, will be used in all other figures and tables.

In a combat game with complete information and perfect weapons, when both red and blue fight optimally, the ensemble of initial states form outcome regions which are never left.[8] That means, contradictory to intuition, there are no role reversals, or change in outcome regions during combat, unless one or the other player makes a fundamental error.

Converting a Dynamic Differential Combat Game to a Discrete Game

The object of differential game theory is to determine the outcome regions and the physical controls required for both aircraft to enable them to stay in each region. In other words, we wish to determine the feedback solution. Applying differential combat game theory to complex dynamic systems including weapons is still beyond the state of the art. However, we know from optimal control theory that, as the number of constraints increase, the problem gets simpler.

Table 1 Outcomes and payoff values for both players

1 Outcome	2 Outcome no.	3 Blue's payoff conservative (maximize)	4 Red's payoff conservative (minimize)
Blue kills red	3	3	3
Draw	2	2	1
Both killed	1	1	2
Red kills blue	0	0	0

Let us try to constrain our combat problem sufficiently to permit a discrete combat game solution without making the result trivial or meaningless. Ordinarily, in an air-combat differential game, the dynamic constraints are the equations of motion of the aircraft and the weapon, and we try to solve for a set of secondary controls such as bank angle and thrust. As we have stated earlier, for realistic aircraft and weapon models, this is beyond the state of the art. However, we do know a lot about effective air-combat maneuvers or guidance laws from pilots' experience as well as from the differential game theory (e.g., pursuit/evasion games). These guidance laws are often intuitively quite reasonable, even though they do not guarantee a specific result. Therefore, instead of considering variables such as bank angle and thrust as controls, we can consider a set of guidance laws as the controls. Then the secondary controls (such as thrust and bank angle) are determined by the guidance laws. The object is to find the best guidance laws among those considered for each initial condition. This results in a discrete combat game, which we know how to solve. The cost of this approach is, of course, that the results are more conservative.

If blue considers using guidance laws ($b_1, b_2,\ldots,b_n$) and red guidance laws ($r_1, r_2,\ldots,r_m$) for each relevant initial condition, we must simulate mn encounters with a specific outcome for each encounter. Then the outcomes can be written as an $n \times m$ matrix, and the minimax theory of discrete games can be used to determine the optimal selection of those guidance laws that were tested. Since there are only four possible outcomes and mn fights, outcomes will often be duplicated in an outcome matrix. Therefore, time may be used as a secondary tag for each outcome, where the loser may choose maximum time and the winner minimum time to the outcome. In terms of optimization for each initial condition, this is equivalent to an exhaustive search over all combinations of guidance laws. For computational economy, it should be said that often not all guidance laws need to be tested for all initial conditions. When it is known a priori that a guidance law is not applicable for a range of initial conditions, the corresponding row or column is then filled in with the least desirable outcome (3 for red and 0 for blue) before evaluating the matrix.

Even though a true combat game solution is as yet not possible, imagine blue had the optimal solution, but he/she does not know if red also has his/her optimal solution. Blue would then attempt to fly undetected to a state that was inside his/her winning region. If successful, he/she would win regardless of whether the opponent used an optimal feedback solution or some other strategy. In the method proposed here, even though it is not optimal, we can use any knowledge we have about the opponent's tactics by simulating his/her guidance laws. If blue knew that red, instead of having the optimal feedback solution, would have very strict rules on how to behave in air combat, or if red was flown automatically with known suboptimal guidance laws, blue could use this information in conjunction with optimal control theory to do even better than the differential game solution. That is, blue's winning region would increase compared to that determined by assuming the opponent plays optimally.

By the above method, we have converted an intractable problem to a tractable one that is easily capable of any degree of improvement, e.g., by considering more exact combat scenarios, aircraft models, weapon models, better guidance laws,

Table 2 Outcomes

Blue's view (maximizer): Blue \ Red	Attack	Evade	Red's view (minimizer): Blue \ Red	Attack	Evade
Attack	1	3	Attack	2	3
Evade	0	2	Evade	0	1

by including the effects of pilot delays, or, known facts of the opponent's tactics.

Simple Example

As a simple example of the method, consider now only two guidance modes for each aircraft. We will call one the attack, and the other the evade mode, to label the intuitive purpose of each guidance mode. Although no added difficulty would occur in the evaluation of the differential game, we will initially use the same attack and evasion modes for both aircraft. By using the same aircraft models for red and blue, we will bring out certain symmetry in the results that are useful for program checkout.

For each set of initial conditions, the payoffs can be represented as a matrix of which an example is given in Table 2. (Note: In the two payoff matrixes, "1" and "2" are reversed as required by Table 1.) The minimax procedure is as follows. If the maximizer (blue) chooses first, he/she chooses the row with the largest minimum, which is "attack" in the example. If both rows had the same largest minimum, "attack" would have been chosen. This fixes the row from which the minimizer (red) will pick the minimum from his/her own matrix. Similarly to the blue case, based on the maxim "offense is the best form of defense," if attack or evasion have the same outcome value, red chooses attack. The outcome for the example is "mutual destruction." If the minimizer chooses first, a similar procedure is followed. In general, for minimax games, the outcomes may depend on the order of choices. As we will see in the results section, for this game, the minimax procedure will make nearly all of the choices identical for both opponents independently of who is choosing first. This is fortunate, since in air-to-air combat we do not have an option of first or second choice. For the few cases where there are different outcomes for different orders of choices, one may follow the order proposed by Kelley.[19]

Dynamic Constraints for the Example

Aircraft and Missile Equations

The F-15 thrust and drag are modeled as functions of speed, throttle setting, and commanded g's. The equations of motion in the plane follow.

$$D = ½\ \rho S v^2 (C_{d0} + C_{brake}) + k_d W^2 n^2 / (½\ \rho S v^2) \tag{1}$$

$$\dot{x} = v \cos\psi \tag{2}$$

$$\dot{y} = v \sin\psi \tag{3}$$

$$\dot{v} = \frac{g}{W}(T - D) \tag{4}$$

$$\dot{\psi} = \text{sgn}(n_c)\frac{g}{v}\sqrt{n^2 - 1} \tag{5}$$

where the subscripts "r" should be attached for the red fighter and "b" for the blue fighter. The engine response is modeled as a 3-s lag between commanded thrust and actual thrust. The structural normal g limit is 7.3 g. For the chosen altitude of 20,000 ft, because of maximum lift limitations, between the corner velocity of Mach 0.59 and the stall speed of Mach 0.4, the normal g limit reduces smoothly from 7.3–3.3 g. For this level-flight condition, the maximum thrust is greater than the drag for all normal g levels to accelerate to a speed slightly above Mach 1 while turning. This is true except for Mach 0.58, where the drag exceeds the maximum, available thrust for a 7.3-g turn. Hence, the speed-control system limits the g level around 0.58 Mach to a lower value when acceleration is required. From Eq. (5), the turn rate is a maximum of 20 deg/s at the corner velocity with an 1800-ft turn radius from

$$R = \frac{v^2}{g\sqrt{n_{max}^2 - 1}} \tag{6}$$

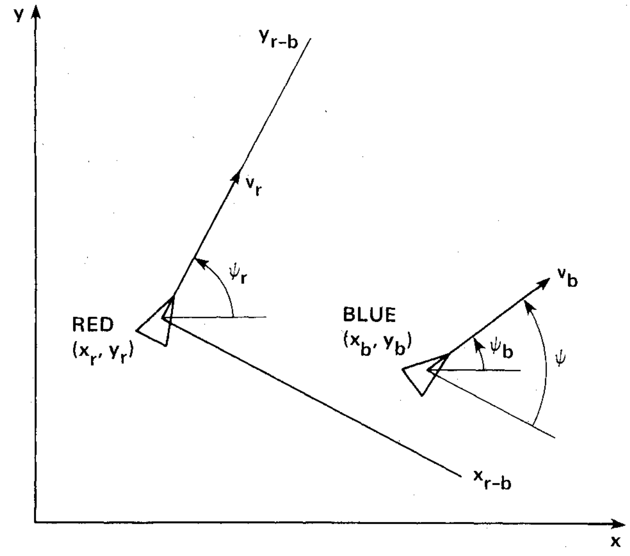


Fig. 1 Conversion to red-centered relative reference frame from flat Earth reference frame.

For the guidance laws and the display, we need the relative reference frames centered on one or the other aircraft (see Fig. 1 for the red fighter).

$$x_{r-b} = (x_b - x_r)\sin\psi_r - (y_b - y_r)\cos\psi_r \tag{7}$$

$$y_{r-b} = (x_b - x_r)\cos\psi_r + (y_b - y_r)\sin\psi_r \tag{8}$$

$$\psi = \psi_b - \psi_r + 90 \text{ deg} \tag{9}$$

The two fighters are armed with short-range missiles. We assume all-aspect missiles of the fire-and-forget type with a kill probability of one, provided they are launched properly. For this simplified example, the firing envelope is defined by a minimum range of 1000 ft and a maximum range of 18,000 ft, with an initial bearing error of less than 40 deg. In order to fire the missile automatically, the opponent must be inside the own missile's firing envelope for 4 s for the seeker to acquire the opponent's aircraft. Once the missile is fired, the opponent will be destroyed after the missile reaches its target. During the flight time of the missile, the opponent can also launch his/her missile, provided all conditions for launch are met. Therefore, one of the possible outcomes of the fight is mutual destruction. The maximum duration of one game is chosen to be 90 s. If neither aircraft has destroyed the other in that time interval, then neither aircraft wins. Hence, there are four possible outcomes: red wins, blue wins, both killed, both survive.

A complete missile simulation is as complex as an aircraft simulation. For the purposes of the game, we are interested only in an approximation of the flight time and a display capability of the missile position. To save computation time, we use a very simple and not very accurate model. The missiles have fixed-speed profiles, and the velocity vectors always point directly at the opponent (ψ_{mr} can change instantaneously). Initial conditions at the moment of firing red's missile,

$$x_{mr} = x_r, \qquad y_{mr} = y_r, \qquad v_{mr} = v_r \tag{10}$$

The missile accelerates at 5 g for the first 5 s during motor burn, and it decelerates at 0.3 g thereafter. The missile intercepts the target when the final range between target and missile is less than the distance covered by the closing speed during one integration interval.

Further Constraints: Guidance Laws

Air-to-air combat is a three-dimensional activity. For this two-dimensional example, we do not have the results from game theoretical studies, or even from pilot experience. Since for the exploration of the new method the example is secondary, we chose the automatic guidance laws somewhat arbitrarily by adapting them from pilot manuals for two dimensions. But we had combat pilots fly against them to test their reasonableness. The details of the manned vs automatic system are described in Ref. 21.

The guidance system of the automatically controlled red fighter has four guidance laws that are selected as a function of the situation. When the range is greater than 3.5 nautical miles and -60 deg$< T_r <$60 deg, a "*proportional navigation,*" also called "*collision bearing*" or "*constant antenna train angle* (*ATA*)" guidance law, is used (see Fig. 2). The rotational rate of the line of sight is

$$\dot{\lambda} = \frac{v_r \sin(\psi_r - \lambda) - v_b \sin(\psi_b - \lambda)}{R} \tag{11}$$

and the guidance law commands the desired turn rate

$$\dot{\psi}_r = K\dot{\lambda}, \qquad K = 2.5 \tag{12}$$

where K is the guidance-law gain. From the equations of motion, this requires a certain number of g to be pulled, limited to the maximum permissible g discussed earlier.

$$n_r = \sqrt{\dot{\psi}_r^2 v_r^2 / g^2 + 1} \tag{13}$$

As the name "constant ATA" implies, if both aircraft would fly at constant speed and heading, a constant antenna train angle would bring the rotation of the line of sight to zero, and the aircraft would be on collision course. Feedback corrects for any changes from constant speed and heading of the opponent. While using this guidance law, we use maximum thrust to quickly reach the enemy.

When red is attacking and the separation is less than 3.5 nautical miles or the inequality -60 deg$< T_r <$60 deg does not hold, a *pure pursuit law* is used, where the red aircraft will attempt to hold a somewhat higher speed than the blue one until the distance between the aircraft is less than 2 n.mi. (see Fig. 3). (Note: this guidance law may be used whether blue attacks or evades.)

Speed command:

$$v_{c_r} = v_b + 50, \qquad R > 2.0 \text{ n.mi.} \tag{14}$$

$$v_{c_r} = v_b, \qquad R \leq 2.0 \text{ n.mi.} \tag{15}$$

A normalized lateral acceleration command, where we have replaced in the linear range T_r with x_r/y_r, is

$$n_{c_r} = k_{nr} x_r / y_r, \qquad -1 \leq n_{c_r} \leq 1,\ y_r > 0,\ k_{nr} = 5 \tag{16}$$

$$n_{c_r} = \text{sgn} x_r, \qquad y_r \leq 0 \tag{17}$$

When, in addition, red is inside a 60 deg cone behind blue, and red points towards blue within a $\pm$40-deg cone, a *tail chase guidance law* is used. Here the commanded airspeed is increased or decreased from the opponent's airspeed as a function of the deviation from a nominal distance.

$$v_{c_r} = v_b + (R - R_{nom})K_{vr} \tag{18}$$

where $R_{nom} = 2$ n.mi. and $K_{vr} = 30$ ft/s/n.mi.

$$n_{c_r} = k_{nr} x_r / y_r, \qquad -1 \leq n_{c_r} \leq 1,\ k_{nr} = 5 \tag{19}$$

The values x_r and y_r can also be estimated as future or past values of the distances to the opponent, which result in lead or lag pursuit.

Speed commands require an *autothrottle*, which is simulated as follows. The thrust command is

$$T_c = D(v_c, n_c) + k_T(v_c - v), \qquad 0 \leq T_c \leq T_{max} \tag{20}$$

where $k_T = 1200$ lb/ft^{-1}/s and D is a feedforward term, which is the expected drag at the commanded speed and bank angle. No integrator term is used, since exact speed control is not critical.

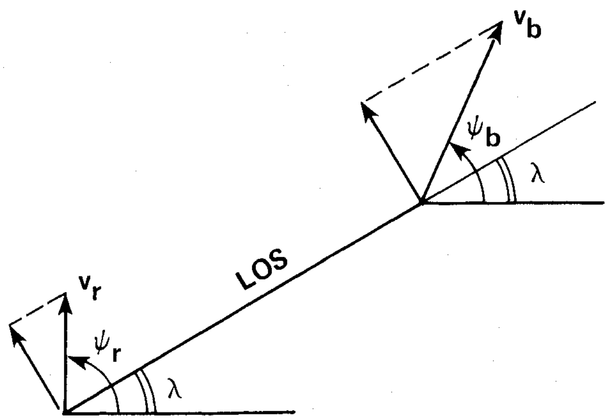


Fig. 2 Diagram for calculating the rotational rate of the line of sight.

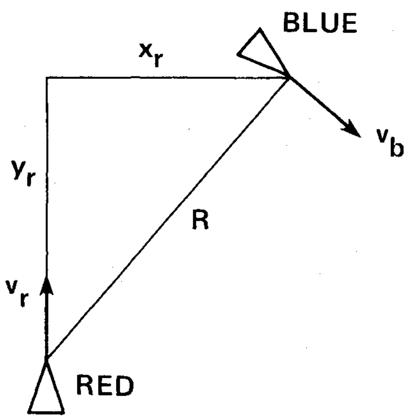


Fig. 3 Diagram for calculating the g command for pure pursuit.

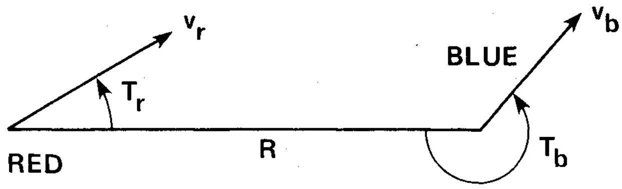


Fig. 4 Coordinate system for initial conditions.

For evasion, there is only one guidance law which is similar to the pure pursuit law. The evader attempts to point the tail of his/her airplane at the attacker and the he/she uses full afterburner thrust, T_{max}, to evade. Such guidance law works only for relatively large distances. Both differential game theory for pursuit/evasion games and pilot-tactics manuals propose complicated last-ditch maneuvers, which are not implemented here. Also, the present guidance laws do not cover other tactics available, such as high-speed single-pass slashing attacks. With the present approach, such tactics could be implemented, and one could determine the initial condition regions where they are successful.

Choice of Initial Conditions

In order to determine zones of different strategies and outcomes, we must choose a state-space representation and quantization of this space into a discrete set of initial conditions to sweep out the significant space. There are also certain symmetries, so that only one-half of the space needs to be covered.

Two investigators, Hague[17] and Kelly[18] chose a representation involving the tracking angles shown in Fig. 4, which we select as two of the necessary variables. For guidance laws which are symmetrical with respect to left and right turns, the flight paths are different as a function of the sign of one of the tracking angles and, therefore, the outcomes of the combat may be different. The symmetry of the game permits removal of mirror symmetrical flight paths. Therefore, one of the ranges of the angles can be reduced to $0 \leq T_b < 180$ deg, while the other must cover the full 360 deg. Also, the range between the opposing aircraft must be considered. To illustrate the need for range as one variable, when two opposing fighters become aware of each other at large ranges, both aircraft will be able to change heading to head-on, ($T_r = T_b = 0$) before entering each other's weapon zones, so that initial headings are not critical for a long-range encounter. However, for small ranges, the outcome will depend critically on the speeds and associated turn radii of the two aircraft as well as on the distance between them and their tracking angles. This will be demonstrated in the results section.

We chose a quantization spacing of 15 deg for heading angles and a quantization spacing in R from 0–30,000 ft, increasing in a geometric series, so that small radii are better covered. In addition, we chose two different initial velocities, $M = 0.68$ and $M = 1.0$. With four flights per initial condition, we must simulate $4 \times 2 \times 13 \times 25 \times 13 = 33{,}800$ fights. This must be repeated for each configuration change of the pair of fighters. In our example, to study encounters between dissimilar aircraft, we reduced the turning capability of the red fighter from 7–6 g, and its maximum thrust by 1500 lb for all speeds. For complete feedback and outcome charts, we would have to investigate unequal initial speeds also. A high initial speed compared to that of the opponent may also require different tactics, such as the slash attack. This is a single attack at high speed followed by an immediate disengagement.

Results

In this section, we examine the results for many encounters to get an insight on the different regions of outcomes and strategies. To permit quick comparison, we present data for the fights between similar and dissimilar aircraft side by side.

Outcome Matrixes

With four trials for each initial condition and four possible outcomes, there are a maximum of 256 different outcome matrixes. For the fights between similar aircraft, only 18 different patterns occur. These patterns, together with the total number of cases occurring, are shown in Table 3. For each case, the appropriate responses for both opponents are shown by encircling one number of the outcome matrix. The cases are arranged in such a way that the symmetry of the game between similar fighters is clearly shown. We can see that when initial conditions are reversed (labeled case "A" and "B") the number of outcomes are the same. We also see

Table 3 Outcome matrixes and frequencies for matched fighters

Case no.	Outcome matrixes b\r: a, r: a	b: a, r: e	b: e, r: a	b: e, r: e	Total no. of cases	Comments
1A	⓪	0	0	2	8	Case-A: Red wins Case-B: Blue wins
1B	③	3	3	2	8	
2A	⓪	2	0	2	380	
2B	③	3	2	2	380	
3A	⓪	3	0	2	1129	
3B	③	3	0	2	1129	
4A	②	2	0	2	81	Stalemate
4B	②	3	2	2	81	
5	②	2	2	2	280	
6	②	3	0	2	1245	
7A	0	2	②	2	168	Attack—Successful evasion Cases A: Blue evades Cases B: Red evades
7B	3	②	2	2	168	
8A	0	3	②	2	8	
8B	3	②	0	2	8	
9A	1	3	②	2	24	
9B	1	②	0	2	24	
10	1	2	2	②	859	Evasion–evasion
11	①	3	0	2	312	Mutual destruction

that for 6292 fights, there are only 312 cases where mutual destruction is the only correct choice. That is, if both attack, both will be killed; and if either one tries to evade, the evader will be killed. There are a few initial conditions, 24 for each opponent, where one of the opponents has a choice of mutual destruction or stalemate, and the other has to accept this or be shot down unilaterally (cases 9A and 9B). For cases 1–3, one missile is fired by the winning combatant. In case 11, there is mutual destruction; two missiles are fired, one by each combatant.

By studying the outcome matrixes of Table 3, we can also answer what happens if one of the opponents makes a mis-

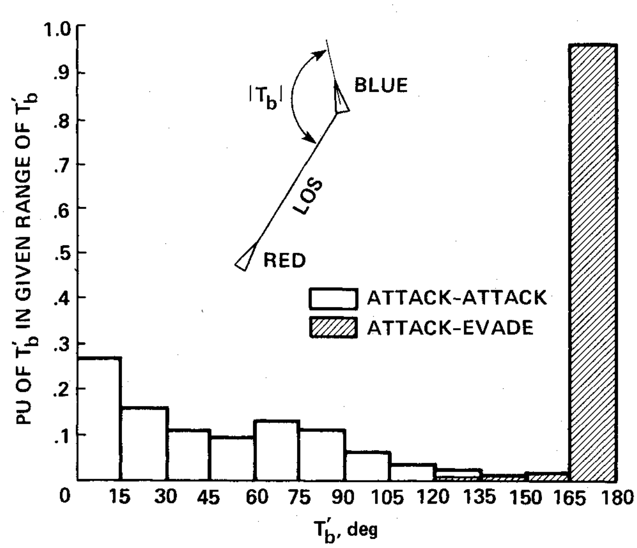


Fig. 5 Histogram of magnitudes of opponent's track angles at the moment of firing.

take. Assume that red makes a mistake with blue responding correctly. Of the 1517 "red wins" cases, 388 become draws and 1129 losses. Of the 2946 "mutual survival" cases (cases 4 through 9), 1534 turn into losses, 24 turn into mutual destruction, and 1388 remain the same. All 312 mutual destruction cases turn into losses. These changes are due to 3181 evasions rather than the appropriate attacks, and to 200 attacks, when evasion would have forced a mutual survival outcome. If a fighter prefers mutual destruction to mutual survival, he/she can force only 24 additional mutual destructions (case 9) on the correctly responding opponent. However, he/she can choose attack instead of evasion (in case 10) for 859 mutual destructions, if the opponent incorrectly attacks back.

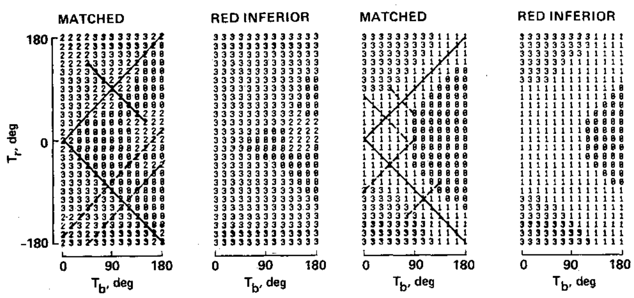
In Fig. 5, two superimposed histograms of the number of missile shots with different track angles for the blue aircraft are shown at the moment that red fires a missile. The track angles have been folded to a 180-deg range by the following method: $T_b' = T_b$ for $T_b < 180$ deg, and $T_b' = 180 - T_b$ for $T_b > 180$ deg. For attack-attack, clearly the head-on shot dominates. (The greater firing opportunity was the reason for designing all-aspect missiles.) Of course, for the losing party, it makes no difference to the outcome, whether the loser attacks or evades. However, if the loser chooses to evade in hopeless situations, we have a preponderance of rear-quarter shots.

In discrete differential games, the outcome may be different depending on which player plays first.[20] This poses a dilemma, since in real air combat such knowledge is not available. Fortunately, in all the 6292 fights between matched fighters, no such cases appear. A negligible number of ambiguous cases appeared for the fights between dissimilar aircraft (4 out of 6292).

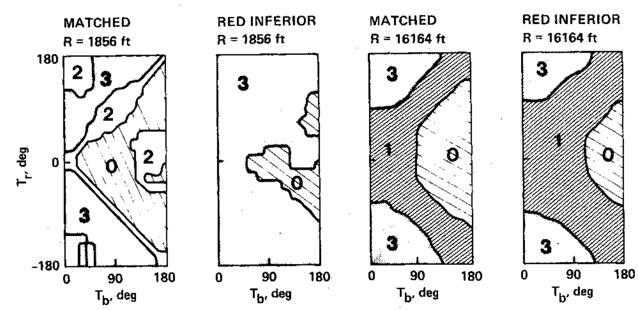
Outcome and Feedback Charts

For the attack-attack situation, Table 4a shows the outcomes for a small initial distance of 1856 ft, and Table 4b for a relatively large distance of 16,164 ft. The heavy-lined diagonals for the fights between similar aircraft indicate angles $|T_r| = |T_b|$ for neutral initial conditions. For the small distance and matched fighters, fights with a neutral initial condition (I.C.) result in stalemate "2," while for the longer distance they end in mutual destruction "1." In the first case, neither fighter can get the advantage and both aircraft perform endless scissoring or circling maneuvers; whereas in the second case, both fighters can turn head-on and fire their weapons. Stalemate for small distances, and mutual destruction for large distances also occur when the initial conditions are close to the diagonal (almost neutral). When the initial

Table 4 Outcomes for attack-attack situation



a) For small distance **b) For large distance**



a) For small distance **b) For large distance**

Fig. 6 Outcome zones for similar and dissimilar fighters for attack-attack situation.

conditions are sufficiently far from neutral, the fighter with the initial advantage wins. The times to the final outcome are generally smaller for the larger distance, because the fight is over quickly in a head-on-encounter, whereas the close-in encounter often results in a lengthy turning fight. The pairs of I.C.'s equidistant from the diagonals have T_r and T_b reversed (some are shown with dashed lines in Table 4). For the encounters between similar aircraft with identical guidance laws, the outcomes for each such pair, therefore, must match if either is "1" or "2" or they must have the opposite outcome, "0" instead of "3" or the reverse. When the initial distance is 0, all fights have exactly neutral I.C. and result in stalemate. When the distance is sufficiently large, all attack-attack fights result in mutual destruction, since both aircraft have sufficient time to turn head-on to the opponent; otherwise, if at least one aircraft evades, all fights result in mutual survival.

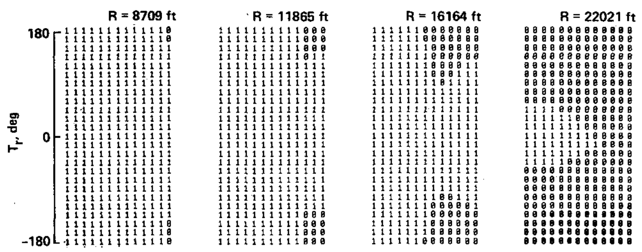
Figure 6 repeats the same data outlining the various outcome regions. This figure permits an easy comparison of encounters between similar and dissimilar aircraft. Blue's advantage is much more evident for the small distance (Fig. 6a), where the win region 3 grows and the draw region 2 is absent. Similarly, for the large distance, red's winning zone 0 shrinks in favor of growth of the mutual destruct zone 1 (Fig. 6b). However, careful comparison of both pairs of figures will show that sometimes a previously winning initial condition for the matched fighters will lead to draws or mutual destruction when blue fights the inferior red. These examples show that the guidance laws are not correct for all initial conditions. For optimal guidance laws, any performance increase in one of the fighters should not cause a disadvantage for that fighter for any initial condition. The reasons for these small losses may be discovered and remedied by studying individual fights, which, from the known initial conditions, can be recorded in detail.

A portion of a feedback table for the red aircraft is shown in Table 5 for the distances where different strategies are needed for different combinations of T_r and T_b, with "0" entries indicating evasion is favorable and "1" entries indicating attack is favorable. For the smaller distances than those shown and at low speeds (Table 5a), which is the usual case for good turning capability, attack is always called for. The reason for this is that there are no successful evasions for ranges smaller than 8708 ft and at any combinations of T_r and T_b. A more sophisticated evasion routine for close-in evasion may partially remedy this situation. Even for high-speed close-in approaches, most combinations of T_r and T_b (not shown) still call for attack. Note that these tables apply only after both fighters are aware of each other. Also notice that the feedback table evolves rather quickly, indicating that our quantization of ranges and speeds are too coarse. Also, the question of how to get to a favorable attack position without being noticed is not addressed.

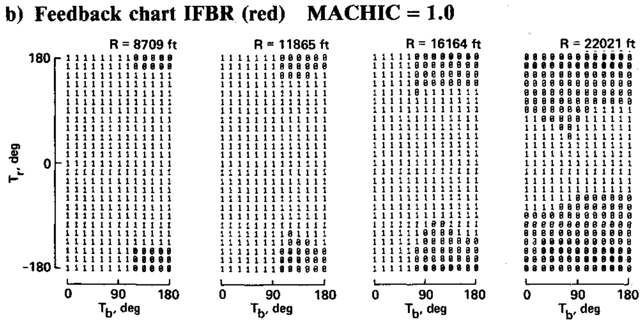
The outcome charts for the low-speed encounters that are given in the feedback table are shown in Fig. 7. Rather than showing a table, for clarity we plotted the outcome regions in order to show how they change with distance. These charts change continuously with range in a complicated manner, and finer range quantization is needed to get sufficient information about the changes. An interesting fact is that the initial contraction of the "2" outcome area, which contracts from 100% at $R = 0$ to a very small value at 1000 ft, then expands and contracts once more, until at 30,000 ft it is again 100%. But, as plotted in Fig. 7, these charts give little insight on how to achieve desirable conditions for attack. More insight is gained by reordering the data as shown in Fig. 8. The outcomes for the minimax plays as well as the tactics are shown as a function of range and T_b with T_r as parameter. There are 25 such figures at a 15-deg interval of T_r. Fig. 8 for $T_r = 0$ is of special interest to the red aircraft. Only one-half of Figs. 8a and 8b are shown, since, for the special case of $T_r = 0$, they are symmetrical with respect to the vertical axis. Assume that red detects blue first and flies toward it to intercept. The figure tells the following: at long range (unless red wants to risk mutual destruction) if blue detects red and begins to react to the threat, it is best to evade. Here, remember that in all air wars 80% of all kills in actual combat were made against unaware opponents. Therefore, a successful evasion is better than a dogfight. However, for a somewhat smaller range, if in addition blue is flying towards red, a fight cannot be avoided and probably will end in mutual destruction. The desirable region to get to before blue will notice red is the region marked "blue gets killed" then blue has little chance of surviving the attack. Figure 8 also shows stalemate outcomes, which somehow have to be broken in a real fight. Comparing Fig. 8a with Fig. 8b, we can see for the disadvantaged red fighter that his/her winning region shrinks, and blue develops a small

Table 5 Portion of a feedback chart

a) Feedback chart IFBR (red) MACHIC = 0.68



b) Feedback chart IFBR (red) MACHIC = 1.0



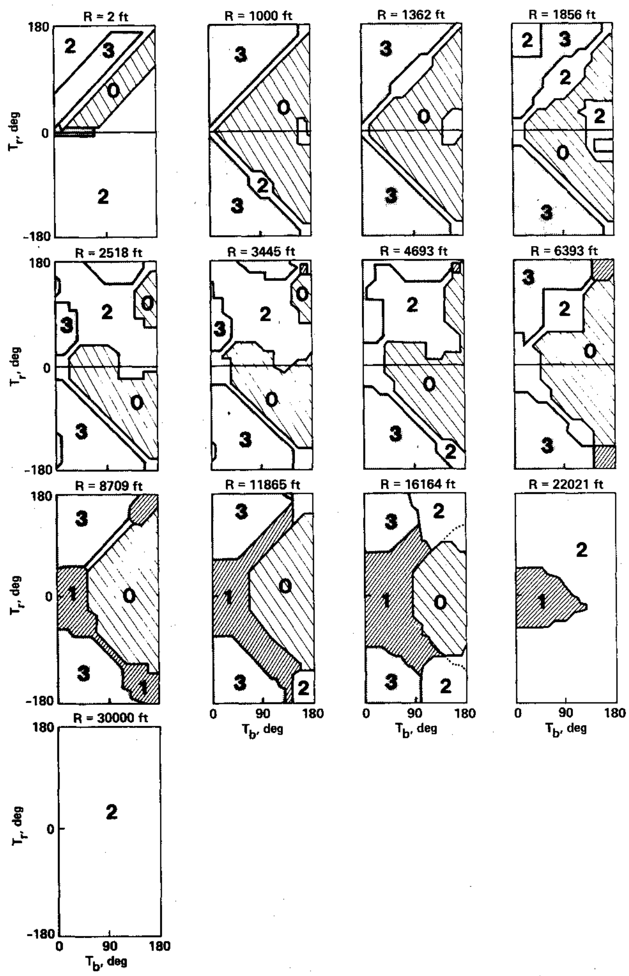
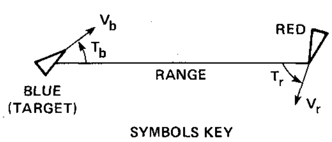
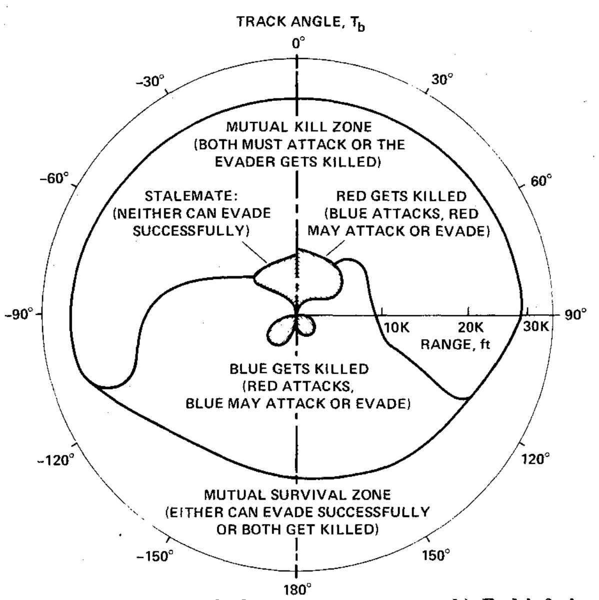


Fig. 7 Outcome charts for low-speed initial conditions $v_r = v_b = 0.68$ M.





a) Red and blue matched b) Red inferior

Fig. 8 Target-centered outcome chart for $T_r = 0$ deg.

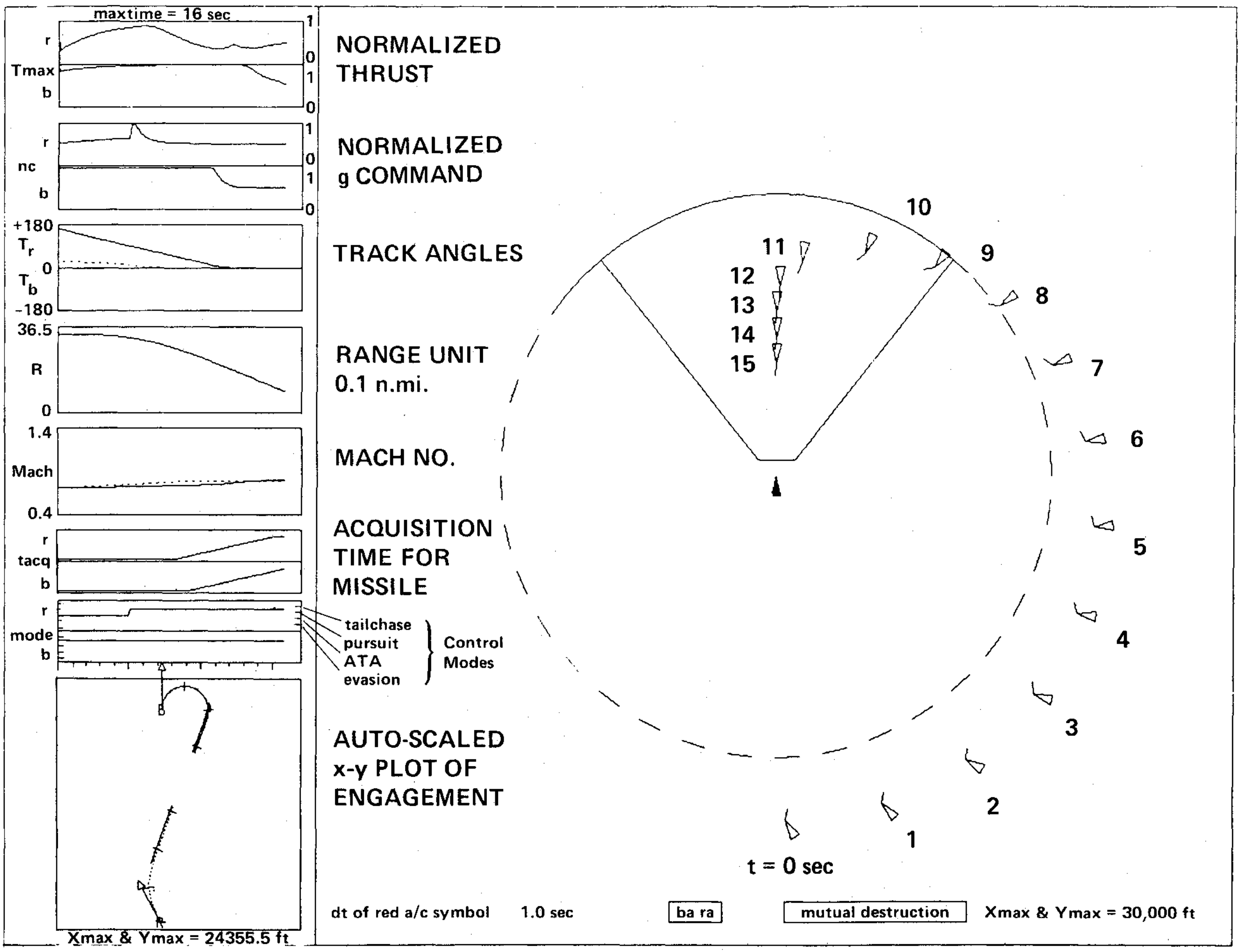


Fig. 9 Postflight analysis display.

winning region compared to the regions for matched fighters in spite of red's initial angular advantage.

Single Encounters

Graphs of detailed fights were essential for pilot debriefing and for checking and improving guidance laws. Since space limitation requires that this section be brief, we give only one detailed sample. Figure 9 shows an attack–attack situation. Both aircraft start relatively far apart, and both attack. Blue has time to turn towards red, who attacked from the back. Red has some initial advantage and keeps his/her track angle approximately equal to 0. Red's missile seeker acquires blue first at about 7 s, and blue's missile seeker acquires red 1.3 s later. This allows red to fire first, but before missile explosion, blue has a chance to fire, resulting in mutual destruction. During the complete engagement, blue was in the *pure pursuit mode,* while red's guidance changes from *ATA* to *pure pursuit.*

Conclusions

A method was proposed to find approximate solutions to complex combat games. For a two-dimensional example, feedback and outcome charts were constructed for opponents that are aware of each other. Essentially, the method permits the selection of the best guidance laws *that were considered* for all possible initial conditions. The feedback and outcome charts can be used to design an air-combat advisory system, or to turn one of the simulated automatic opponents into an intelligent opponent against a human pilot. The outcome charts may also be used in earlier phases of air combat, not covered in this study, by providing insight into the enemy's vulnerable areas. In order to complete the feedback charts required for this example, charts for initial conditions with different initial speeds will have to be added, and additional tactics from those given here will have to be explored. Although the method, which is based to a large extent on simulation, is very computation intensive, it has the advantage that it can use accurate models of aircraft, weapons, and tactics. The method permits incorporation of pilot-suggested tactics as well as tactics obtained from differential-game analysis. The simulation will automatically reject "optimal" tactics obtained from simplified analysis in regions of the initial-condition space where such tactics are unsuccessful. The approach explored here can be extended to three dimensions.

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